Derivation and Calculation of the Reflection Coefficient for the Single-Slot LCX

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Abstract

In order to suppressing the resonance point of leaky coaxial cable, first we need to calculate the reflection coefficient under the single slot, thereby obtaining a total reflection coefficient of the entire cable. Detailed analysis of the reflection in the case of a single slot, and deduce the reflection coefficient of the single-slot, specific examples are given calculation result and analysis. Finally, there is a simple analysis for the derivation of the total reflection coefficient of the leaky coaxial cable.

1 Introduction

Leaky coaxial cable is usually called LCX for short. It has both the transmission and radiation characteristics, and recently it is widely used to solve information transmission problems in the areas where the wireless communication can not work well, such as railway, tunnel and underground store and so on. But nowadays along with the increase of communication period and the frequency band, leaky coaxial cable can not meet the need of larger and larger information transmission capacity, so it is a trend to extend the frequency band of LCX[1].One way is to inhibit the resonance points. If you make the amplitude of the reflection coefficient in the expansion of the band reduced to communication within the allowable range, these frequencies will be suppressed interference, which can achieve the purpose of the extended band. The key point is to calculate the total reflection coefficient of the leaky coaxial cable, to reaches the purpose of the extended band[2]. Literature [1, 2, 3, 4] studied how to suppress the resonance point. However, these papers have focused on how to suppress the resonance point. They have not shown methods to derivation and calculation of the total reflection coefficient of LCX. According to the theory of small reflection[5] ,we can know that, as long as the total reflection coefficient can be obtained to calculate the single-slot overall

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reflection coefficient. Single-slot of leaky coaxial cable is the basic unit of the leaky coaxial cable, plays a crucial role in the parameters for calculating the leaky coaxial cable. In this paper, the detailed process of the specific calculation of the reflection coefficient of the single-slot of leaky coaxial cable, derivation and case count results provide a theoretical basis for the design of the leaky coaxial cable.

2 Single-slot LCX model

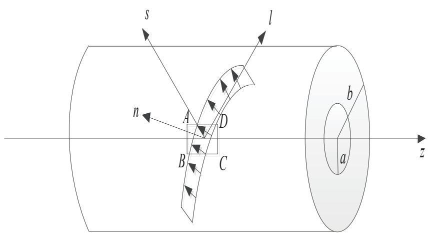

Fig.1 a single slot of leaky coaxial cable schematic

Figure 1 shows a single slot in the leaky coaxial cable. Set the coaxial cable outer surface of the radius *b*, inner conductor radius *a*, the slot width *w* and the length $2l_0$, the slot for the z-axis angle ξ. The slot can be divided into finite small parts, each part can be approximated on a plane, each part shown in Figure 2.

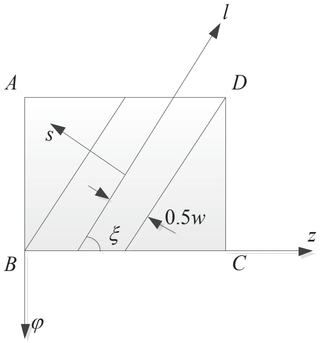

Fig.2 part of the schematic

From figure 2 part of the schematic , we can get sl coordinate system can get from zφ coordinate system by coordinate transformation. Hence

$$z = l\cos\xi - s\sin\xi \qquad (1)$$

3 Derivation the reflection coefficient for the single-slot LCX

Assume $\overrightarrow{E_1}$, $\overrightarrow{H_1}$ are the slot electric and magnetic field vector. $\overrightarrow{E_2}$, $\overrightarrow{H_2}$ are the electric and magnetic field vector of the coaxial leaky cables to the +z or -z direction propagation of the TEM wave.

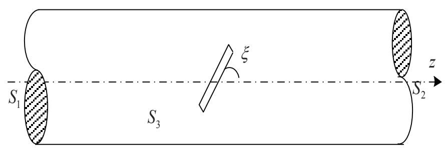

Fig. 3 single slot leaky coaxial cable analysis schematic

To consider the closed surface by the composition of $S = S_1 + S_2 + S_3$,shown in Figure 3 single slot leaky coaxial cable analysis schematic ,which S_1 , S_2 for the two reference surface of the slot at both ends of the coaxial cable from the slot far enough, the only TEM mode the main component, S_3 is the coaxial leaky cables internal surface between S_1 and S_2 .S surface passive, according to the reciprocity theorem can be obtained

$$\int_s \left(\overrightarrow{E_1}\times\overrightarrow{H_1} - \overrightarrow{E_2}\times\overrightarrow{H_2}\right) d\vec{S} = 0 \qquad (2)$$

Since the width of the slot is very thin, so we can assume that the electrical field on the slot aperture is only changing in the l-direction and is not changing in the s-direction, therefore the electrical field E can be decomposed into two components $\overrightarrow{E_z}$ and $\overrightarrow{E_\varphi}$.

We can get the electrical field[6]

$$\overrightarrow{E(l)} = \frac{2D_0}{w\,\psi \sin(2kl_0)}\{\sin\left[(\beta' - k)l_0\right]\exp(jkl)$$

$$-\sin\left[(\beta' + k)l_0\right]\exp(-jkl)$$

$$+\sin(2kl_0)\exp(-j\beta' l)\} \qquad (3)$$

where

$$D_0 = -(0.5j\omega\mu_0 H_0 \sin\theta)/(k_0^{\,2} - \beta'^2)$$

$$H_0 = \frac{V_0\sqrt{\varepsilon_r}}{r_b\sqrt{\mu_0}\ln(b/a)}$$

$$\beta' = \beta\cos\theta$$

$$\psi = \int_{-l_0}^{l_0}\frac{e^{-jk_0R}}{2\pi R}dl, R = \sqrt{l^2 + (0.5w)^2}$$

l_0 is the half length of the slot.
w is the width of the slot.
We can get the electric and magnetic field expression of coaxial cable[7]

$$\overrightarrow{E_2} = \frac{V_0}{\ln(b/a)}\frac{\overrightarrow{a_r}}{r}e^{-jk_0z} \qquad (4)$$

$$\overrightarrow{H_2} = \frac{V_0}{Z\ln(b/a)}\frac{\overrightarrow{a_\varphi}}{r}e^{-jk_0z} \qquad (5)$$

Where Z is the wave impedance between the outer conductor and inner conductor

By the expression (3) shows that $\overrightarrow{E_2}$ only have r-direction , so it contribution to the integral on S_3 is zero. $\overrightarrow{E_1}$ can be decomposed into z-direction and ϕ-direction , $\overrightarrow{H_2}$ is ϕ-direction , it contribution to the integral on S_1 and S_2 is zero. Hence we can get the new equation of the equation (2)

$$\int_{S_{slot}}\left(\overrightarrow{E_1}\times\overrightarrow{H_2}\right)d\vec{S} - \int_{S_1+S_2}\left(\overrightarrow{E_2}\times\overrightarrow{H_1} - \overrightarrow{E_1}\times\overrightarrow{H_2}\right)d\vec{S}$$

$$= I_1 + I_2$$

Where I_1 and I_2 separately are the integral on S_1 and the integral on S_2. Tangential component of the reflected wave of $\overrightarrow{E_1}$, $\overrightarrow{H_1}$ can contribute to the integration on S_1 are $\overrightarrow{E_{1t}}$ and $\overrightarrow{H_{1t}}$, and we can assume that $\overrightarrow{E_{1t}} = R\overrightarrow{E_t}e^{j\beta z}$, $\overrightarrow{H_{1t}} = -R\overrightarrow{H_t}e^{j\beta z}$.

Where $\overrightarrow{E_t}$, $\overrightarrow{H_t}$ are the tangential component of TEM , β is propagation constant , R expressed along the z-axis positive or negative to the transmission of electromagnetic waves scattering coefficient. Tangential component of the transmission wave of $\overrightarrow{E_1}$, $\overrightarrow{H_1}$ can contribute to the integration on S_2 are $\overrightarrow{E_{1t}}$ and $\overrightarrow{H_{1t}}$, and we can assume that $\overrightarrow{E_{1t}} = T\overrightarrow{E_t}e^{-j\beta z}$, $\overrightarrow{H_{1t}} = T\overrightarrow{H_t}e^{-j\beta z}$

Hence we can get

$$I_1 = \int_{S_1}\left(\overrightarrow{E_2}\times\overrightarrow{H_1} - \overrightarrow{E_1}\times\overrightarrow{H_2}\right)d\vec{S}$$

$$= \int_{S_1}\begin{pmatrix}\overrightarrow{E_t}e^{-j\beta z_1}\times R\overrightarrow{H_t}e^{j\beta z_1}\\ +R\overrightarrow{E_t}e^{j\beta z_1}\times\overrightarrow{H_t}e^{-j\beta z_1}\end{pmatrix}\vec{z}d\vec{S}$$

$$= 2R\int_{S_1}\left(\overrightarrow{E_t}\times\overrightarrow{H_t}\right)\vec{z}d\vec{S}$$

$$I_1 = \int_{S_1}\left(\overrightarrow{E_2}\times\overrightarrow{H_1} - \overrightarrow{E_1}\times\overrightarrow{H_2}\right)d\vec{S}$$

$$= \int_{S_1}\begin{pmatrix}\overrightarrow{E_t}e^{-j\beta z_1}\times T\overrightarrow{H_t}e^{-j\beta z_1}\\ -T\overrightarrow{E_t}e^{-j\beta z_1}\times\overrightarrow{H_t}e^{-j\beta z_1}\end{pmatrix}\vec{z}d\vec{S}$$

$$= 0$$

$$R = \frac{\int\limits_{S_{slot}} \left(\overrightarrow{E_1} \times \overrightarrow{H_2}\right) d\vec{S}}{2\int\limits_{S_1} \left(\overrightarrow{E_t} \times \overrightarrow{H_t}\right) \vec{z} d\vec{S}} \tag{6}$$

Since the transmission line with the TEM wave electric and magnetic fields are associated with a single value of the voltage wave and current wave. R is the transmission line reflection coefficient. We can get the equation by the equation (1) (3) and the equation (4) (5) into the equation (6).

$$\begin{aligned}
&\int\limits_{S_{slot}} \left(\overrightarrow{E_1} \times \overrightarrow{H_2}\right) d\vec{S} \\
&= \int\limits_{S_{slot}} E(l)\cos\xi \frac{V_0}{Z\ln(b/a)} \frac{1}{b} e^{-j\beta z} dS \\
&= \int\limits_{-w/2}^{w/2} \frac{1}{w} e^{j\beta s \sin\xi} \int\limits_{-l}^{l} M(l)\cos\xi \frac{V_0}{Z\ln(b/a)} \frac{1}{b} e^{-j\beta l\cos\xi} dl \\
&= \frac{\sin(\beta w/2\sin\xi)}{\beta w/2\sin\xi} \int\limits_{-l}^{l} M(l)\cos\xi \frac{V_0}{Z\ln(b/a)} \frac{1}{b} e^{-j\beta l\cos\xi} dl
\end{aligned}$$

$$\begin{aligned}
&2\int\limits_{S_1} \left(\overrightarrow{E_t} \times \overrightarrow{H_t}\right) \vec{z} d\vec{S} \\
&= 2\int\limits_{S_1} \frac{V_0}{\ln(b/a)} \frac{1}{r} \frac{V_0}{Z\ln(b/a)} \frac{1}{r} dS \\
&= 2\int_a^b dr \int_0^{2\pi} \frac{V_0}{\ln(b/a)} \frac{1}{r} \frac{V_0}{Z\ln(b/a)} d\varphi
\end{aligned}$$

We can get the reflection coefficient expression of the single-slot LCX that

$$R = \frac{\sin(\beta w/2\sin\xi)}{2\pi V_0 \beta w b} \int\limits_{-l}^{l} M(l) e^{-j\beta(l\cos\xi)} dl$$

Where $M(l) = w\, E_s(l)$

We can see from the expression of the reflection coefficient for the single-slot LCX that it have a close relationship with the slot length l_0 ,slot width *w*, inclination angle ξ, and the angular frequency ω. The following are specific calculations to analysis their relationship.

4 Calculation of the reflection coefficient for the single-slot LCX

When b=20.65mm, Z=50, epsir=1.26, l_0 =16.2mm, w=3.3mm, V0=1, f=900 MHz. We can get Figure 5 the change of reflection coefficient with the change of ξ (sita).

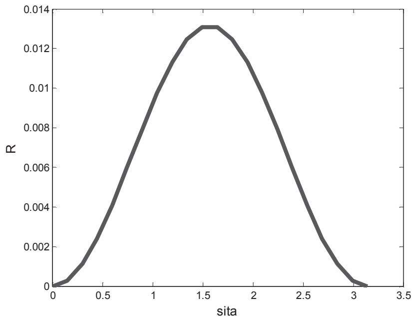

Fig.4 the change of reflection coefficient with the change of ξ

From the figure 4, we can get that the reflection coefficient is maximum at $\xi = \pi/2$. And we can get the value from the figure that $|R| = 0.0131$. Which is same to the literature [6] about the reflection coefficient by obtained from an approximate theoretical analysis at frequency of 900 MHz[6].

When b=20.65mm, Z=50, epsir=1.26, l_0 =16.2mm, w=3.3mm, V0=1, f=900 MHz. We can get Figure 5 that the change of reflection coefficient with the change of frequent. From the figure 5 we can get that the reflection coefficient is increased with increasing frequency. As the frequency increases, its wavelength is closer and closer to the length of the slot, arising more and more obvious reflection. Similarly, in

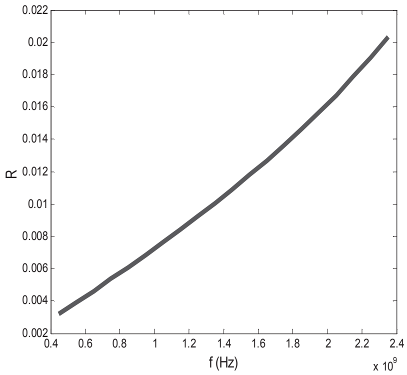

Fig.5 the change of reflection coefficient with the change of frequent the case of the frequency constant, as the slot length increases, the reflection coefficient will be gradually increased, the actual computation result is indeed the case. However, the resonant characteristic of the single-slot leaky coaxial cable does not reflect the resonance characteristics of the leaky coaxial cable.

5 Analysis of the total resonance point of the LCX

We can get from the small reflection theory that when the value of the modulus of the reflection coefficient of each slot than 1 small lot, the total reflection coefficient can be a good approximation that

$$\Gamma_{total} = \Gamma_1 + \Gamma_2 e^{-2j\theta} + \Gamma_3 e^{-4j\theta} + \cdots + \Gamma_N e^{-2(N-1)j\theta} \quad (7)$$

Where Γ_1 is the modulus value of the reflection coefficient of the first slot along the z-axis positive. Γ_N is the modulus value of the reflection coefficient of the N-th slot along the z-axis positive. θ is the electrical length of adjacent slots. When $e^{-2j\theta} = 1$, $4\pi \frac{f}{f_0} = 2k\pi$ where $f_0 = \frac{c}{\sqrt{\varepsilon_r} P}$, the values of total reflectance coefficient of these frequency points will be maximum.

However, when use the value of the reflectance coefficient for signal-slot LCX at frequency of 900 MHz into the equation (7), and the number of cycles is above 100, the total reflection coefficient will above 1.Which is does not match the theory of reflectance coefficient. Therefore, the equation (7) is not applicable for the number of slots is abundant. We need to get a more precise theoretical result.

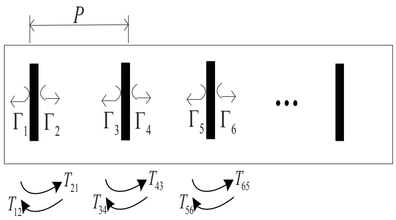

Fig.6 Equivalent analysis for the leaky coaxial cable

The whole cable can be equivalent to a series junction as shown in fig.6. When the incident wave strikes the first junction, a partial reflected wave of amplitude Γ_1 is produced. A transmitted wave of amplitude T_{21} is then incident on the second junction. A portion of this is reflected to give a wave of amplitude $T_{21}\Gamma_3 e^{-2j\theta}$ incident from the right on the first junction. A portion $T_{12}T_{21}\Gamma_3 e^{-2j\theta}$ is transmitted, and a portion $T_{12}T_{21}\Gamma_3\Gamma_2 e^{-2j\theta}$ is reflected back toward second junction[5]. However, taking into account the reflection coefficient calculation quantity and the value of refection coefficient for signal-slot LCX, the higher powers of the part can be negligible. Considering the value of refection coefficient for signal-slot LCX at frequency of 900 MHz, the number of slot more than 1000 will contribution to the total reflection coefficient. Therefore each slots only once contribution to the total coefficient reflection. According to the analysis of the above two slots, we can obtain the total reflection coefficient.

$$\Gamma_{total} = \Gamma_1 + T_{12}T_{21}\Gamma_3 e^{-2j\theta} + T_{21}T_{43}\Gamma_5 T_{34} e^{-2j\theta} T_{12} e^{-2j\theta} + \cdots$$

where $\Gamma_1 = \Gamma_3 = \Gamma_5 = -\Gamma_2 = -\Gamma_4 = -\Gamma_5$

$$T_{21} = T_{43} = T_{65} = -T_{12} = -T_{34} = -T_{56}$$

So, we can get that

$$\Gamma_{total} = \Gamma_1 \frac{1 - T_{21}^{2N} e^{-j2N\theta}}{1 - T_{21}^{2} e^{-j2\theta}} \qquad (8)$$

We can get the relationship of total reflection coefficient with frequency when use the reflection coefficient for the signal-slot LCX into the total reflection coefficient. We can know from the equation (8) that the resonance point appears at $f = 0.5kf_0$ （$f_0 = \frac{c}{\sqrt{\varepsilon_r} P}$, k=0,1,2…）. As the frequency increases, the reflection coefficient for the signal-slot LCX is gradually increased, and the total reflection coefficient will gradually increase at the resonance point. The paper [1,2,3,4] have detailed description of how to suppress the resonance point.

6 Conclusion

In this paper, detailed derivation of the analytical expression of the reflection coefficient for signal-slot LCX, and example count and analysis the expression. We can get that with the increase of the inclination angle of slot, the reflection coefficient is gradually increased, the maximum at the perpendicular to the leaky coaxial cable, but as the frequency increases, the reflection coefficient is also increased, and these are consistent with the practical. With the increase in the width or the length of slots, the reflection coefficient is also increasing. Secondly, theoretical analysis and derivation of the follow-up to the total reflection coefficient, deduced the total reflection coefficient expression, whose error is relatively small. Providing a theoretical basis for the further design of leaky coaxial cable.

7 References

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