Determination of Radioactivity of Mixed Samples by Analysis of Gamma Ray Spectra

by Tatsusaburo SUZUKI*・Kazuo GODA**
Noboru SUZUKI***・Hideki OHNO****

A study of the determination of the activity of each nuclide of mixed samples is carried out by utilization of a NaI (Tl) scintillator. By the analysis of the combined full energy peaks of gamma ray spectra, the values of the radioactivity of each nuclide are determined. For the mixed samples of ^{137}Cs and ^{95}Zr, the results have good agreements with that of the mixed samples.

I. Introduction

An analysis of the intensity of two closed gamma ray spectra with a NaI (Tl) scintillator was studied by T. Suzuki, K. Goda and N. Suzuki(1),(2).

In the present study the mixed samples of ^{137}Cs ($h\nu_0=661.65$ keV) and ^{95}Zr ($h\nu_1=724.18$ keV, $h\nu_2=756.72$ keV) are used. By the measurement of the total peak area and the full width at half maximum, at one 3rd maximum or at one 4th maximum of the combined full energy peaks, the number of gamma ray photons emitted in unit time N_0 (for $h\nu_0$), N_1 (for $h\nu_1$) or N_2 (for $h\nu_2$) is determined. From the branch ratio of each gamma ray photons the radioactivity of each nuclide may be decided.

II. Calculations

1. Standard Sources

Standard sources of ^{137}Cs and ^{54}Mn shown in Table 1 are measured with a NaI (Tl) scintillator of 1 3/4"$\phi\times$2" size at a distance of 1.7 cm from the surface of the detector. The measured values of peak efficiency and FWHM are also shown in the Table.

2. Mixed Samples

Three mixed samples are measured. The radioactivity of each nuclide of the samples is shown in Table 2. Peak efficiency and FWHM of the monochromatic gamma rays emitted from the mixed samples are obtained by interpolation of the values of Table 1 and are shown in Table 3.

---

*いわき明星大学学長　放射線物理学
**理工学部物理学科助教授　放射線物理学
***理工学部物理学科助手　宇宙線
****大学院博士課程、元高分解能分析電子顕微鏡センター嘱託助手

Table 1 Peak efficiency and FWHM of gamma ray standard sources

Nuclide (Activity)	Gamma ray energy (keV)	Branch ratio (%)	Peak efficiency	FWHM (keV)
^{137}Cs (2236Bq)	661.65	85.0	0.01653	60
^{54}Mn (706.7Bq)	834.83	99.98	0.01181	70

Table 2 Radioactivity of nuclide of mixed samples

	^{137}Cs	^{95}Zr
Sample I	2236 Bq	1780 Bq
Sample II	2236 Bq	1190 Bq
Sample III	2236 Bq	950 Bq

Table 3 Branch ratio, peak efficiency and FWHM of the monochromatic gamma photons emitted from mixed samples

Nuclide	Gamma ray energy	Branch ratio	Peak efficiency	FWHM
^{137}Cs	$h\nu_0=661.65$ keV	$b_0=0.850$	$\eta_0=0.01653$	$w_0=60$ keV
^{95}Zr	$h\nu_1=724.18$ keV	$b_1=0.431$	$\eta_1=0.01450$	$w_1=64$ keV
	$h\nu_2=765.72$ keV	$b_2=0.546$	$\eta_2=0.01361$	$w_2=65.4$ keV

3. Calculation Method

As the full energy peak of monochromatic gamma rays measured with a NaI (Tl) scintillator is expressed approximately by the Gaussian distribution[1], each peak may be written as

$$p_0(x)=\frac{A}{a_0\sqrt{\pi}}\exp\left[-\frac{(E-h\nu_0)^2}{a_0^2}\right], \tag{1}$$

$$p_1(x)=\frac{\alpha_1 A}{a_1\sqrt{\pi}}\exp\left[-\frac{(E-h\nu_1)^2}{a_1^2}\right], \tag{2}$$

or

$$p_2(x)=\frac{\alpha_2 A}{a_2\sqrt{\pi}}\exp\left[-\frac{(E-h\nu_2)^2}{a_2^2}\right], \tag{3}$$

where A (for $h\nu_0$), $\alpha_1 A$ (for $h\nu_1$) or $\alpha_2 A$ (for $h\nu_2$) is the peak area and a_i is given by w_i (FWHM) of the monochromatic gamma radiation of energy $h\nu_i$ ($i=0$, 1, 2) as

$$a_i=\frac{w_i}{2\sqrt{\ln 2}}\ (i=0,\ 1,\ 2). \tag{4}$$

Assuming the peak area from $h\nu_1$ and $h\nu_2$ of the gamma photons from ^{95}Zr is αA, the values α_1 and α_2 in Eq. (2) and Eq. (3) are expressed from the values of Table 3 as

$$\alpha_1=\frac{b_1\eta_1}{b_1\eta_1+b_2\eta_2}\alpha=0.4569\alpha \tag{5}$$

and

$$\alpha_2=\frac{b_2\eta_2}{b_1\eta_1+b_2\eta_2}\alpha=0.5431\alpha. \tag{6}$$

The combined full energy peaks of ^{137}Cs ($h\nu_0$) and ^{95}Zr ($h\nu_1$ and $h\nu_2$) are expressed as

$$p_c(E)=\frac{A}{a_0\sqrt{\pi}}\exp\left[-\frac{(E-h\nu_0)^2}{a_0^2}\right]+\frac{\alpha_1 A}{a_1\sqrt{\pi}}\exp\left[-\frac{(E-h\nu_1)^2}{a_1^2}\right] +\frac{\alpha_2 A}{a_2\sqrt{\pi}}\exp\left[-\frac{(E-h\nu_2)^2}{a_2^2}\right], \tag{7}$$

or

$$T(x)=\frac{a_0\sqrt{\pi}}{A}p_c(E)=\exp(-x^2)+\alpha_1\frac{w_0}{w_1}\exp\left[-\left(\frac{w_0}{w_1}x-\delta_1\right)^2\right] +\alpha_2\frac{w_0}{w_2}\exp\left[-\left(\frac{w_0}{w_2}x-\delta_2\right)^2\right], \tag{8}$$

where

$$x=\frac{E-h\nu_0}{a_0}$$
$$\delta_1=\frac{2\sqrt{\ln 2}}{w_1}(h\nu_1-h\nu_0) \tag{9}$$
$$\delta_2=\frac{2\sqrt{\ln 2}}{w_2}(h\nu_2-h\nu_0).$$

4. Calculated Results

For the given value of α, the combined peaks of Eq. (7) or Eq. (8) are computed and are shown in Fig. 1. Also for the given intensity of spectrum the values

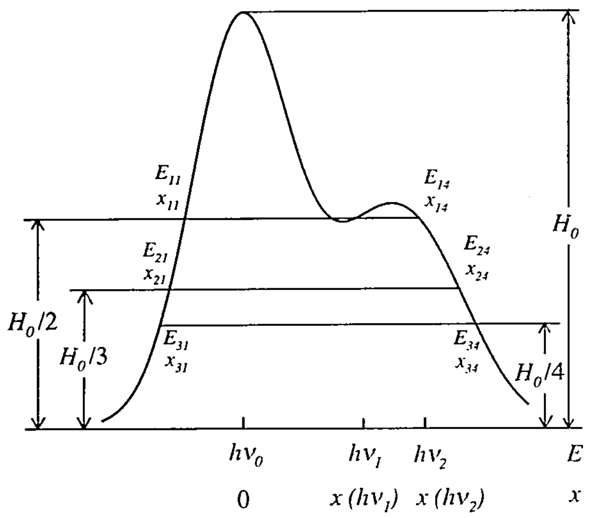

Fig.1 Combined full energy peaks calculated from Eqs. (7) and (8)

Table 4 Calculated values of $x_i \equiv x_{i4} - x_{i1} (i=1, 2, 3)$ from Eqs. (5), (6), (8) and (9) for the given values of α

α	$x_1 \equiv x_{14} - x_{11}$	$x_2 \equiv x_{24} - x_{21}$	$x_3 \equiv x_{34} - x_{31}$
0.36			3.83212
0.38			3.92623
0.40			4.00225
0.42			4.06628
0.44			4.12193
0.46		3.59711	4.17097
0.48		3.69071	
0.50		3.76494	
0.52		3.82687	
0.54		3.88052	
0.56		3.92797	
0.72	3.44999		
0.74	3.50294		
0.76	3.54899		
0.78	3.59005		
0.80	3.62701		

$x_1 \equiv x_{14} - x_{11}$ (for half maximum),

$x_2 \equiv x_{24} - x_{21}$ (for one 3rd maximum)

or

$x_3 \equiv x_{34} - x_{31}$ (for one 4th maximum)

in Fig. 1 are calculated from Eqs. (5), (6), (8) and (9) and are shown in Table 4.

III. Expeiment

The mixed samples of Table 2 are measuered with the same scintillator and the same geometric arrangement of the standard sources. The measured combined full energy peaks are shown in Fig. 2.

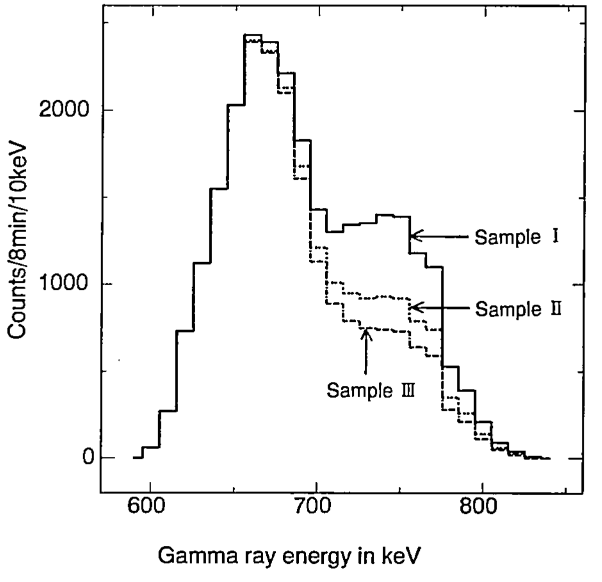

Fig. 2 Combined full energy peaks of mixed samples measured by a NaI (Tl) scintillator of 1 3/4″×2″ size at a distance of 1.7 cm from surface of detector

IV. Results and Discussion

For the mixed samples the total areas and the values $[E_{i4} - E_{i1}]_{exp}$ (i=1, 2, 3) of the combined full energy peaks are obtained from Fig. 2 and are shown in Table 5.

Results of Sample I

From Eqs. (4), (9), Table 3 and Table 5 the value x_1 is obtained as

$$[x_1]_{exp} = [x_{14} - x_{11}]_{exp} = \frac{1}{a_0}[E_{14} - E_{11}]_{exp}$$

$$= \frac{2\sqrt{\ln 2} \times 128\text{keV}}{w_0} = 3.55223. \qquad (10)$$

Table 5 Total area and full width of combined full energy peaks of mixed samples obtained by experiment

	Total area $A+\alpha_1 A+\alpha_2 A$ $=A+\alpha A$ Counts/8min	Full width $E_{i4}-E_{i1}$ $(i=1, 2, 3)$
Sample I	26380	$[E_{14}-E_{11}]_{exp}=128$ keV
Sample II	22630	$[E_{24}-E_{21}]_{exp}=137$ keV
Sample III	21120	$[E_{34}-E_{31}]_{exp}=144$ keV

From Table 4 the value α is obtained as

$$\alpha=0.76158. \tag{11}$$

The values of α_2 and α_3 may be written from Eqs. (5) and (6) as

$$\begin{aligned}\alpha_1&=0.3480\\ \alpha_2&=0.4136.\end{aligned} \tag{12}$$

The average peak efficiency of combined full energy peaks of Sample I is given as

$$\langle\eta\rangle=\frac{\eta_0+\alpha_1\eta_1+\alpha_2\eta_2}{1+\alpha_1+\alpha_2}=\frac{0.027205}{1.76158}=0.01544. \tag{13}$$

From Table 5 the number of photons emitted from Sample I in unit time is obtained as

$$N_0+N_1+N_2=\frac{26380}{0.01544}/8\text{min}=3559\ \text{s}^{-1}. \tag{14}$$

As N_1 or N_2 is given[1] as

$$\begin{aligned}N_1&=\frac{\alpha_1\eta_0}{\eta_1}N_0=0.3967N_0\\ N_2&=\frac{\alpha_2\eta_0}{\eta_2}N_0=0.5023N_0,\end{aligned} \tag{15}$$

from Eqs. (14) and (15)

$$1.899N_0=3559\ \text{s}^{-1} \tag{16}$$

or

$$N_0=1874\ \text{s}^{-1}. \tag{17}$$

From Table 3 the radioactivity of ^{137}Cs of Sample I is obtained as

$$A_{\mathrm{I}}(^{137}\mathrm{Cs})=\frac{N_0}{b_0}=2205\ \mathrm{Bq}. \tag{18}$$

From Eqs. (15), (17) and Table 3 the radioactivity of ^{95}Zr of Sample I is obtained as

$$A_{\mathrm{I}}(^{95}\mathrm{Zr})=\frac{N_1}{b_1}=\frac{0.3967N_0}{0.431}=1725\ \mathrm{Bq} \tag{19}$$

or

$$A_{\mathrm{I}}(^{95}\mathrm{Zr})=\frac{N_2}{b_2}=\frac{0.5023N_0}{0.546}=1724\ \mathrm{Bq}. \tag{20}$$

Results of Sample II and Sample III

In the same process of Sample I, the values of α of Sample II is obtained as $\alpha=0.5119554$ and the radioactivities of ^{137}Cs and ^{95}Zr of Sample II are obtained as

$$A_{\mathrm{II}}(^{137}\mathrm{Cs})=2210\ \mathrm{Bq} \tag{21}$$

and

$$A_{\mathrm{II}}(^{95}\mathrm{Zr})=1160\ \mathrm{Bq}. \tag{22}$$

The value of α of Sample III is obtained as $\alpha=0.398415$ and the radioactivities of ^{137}Cs and ^{95}Zr of Sample III are obtined as

$$A_{\mathrm{III}}(^{137}\mathrm{Cs})=2226\ \mathrm{Bq} \tag{23}$$

and

$$A_{\mathrm{III}}(^{95}\mathrm{Zr})=911\ \mathrm{Bq}. \tag{24}$$

The values of radioactivity in Eqs. (18)～(24) obtained by the present study agree with the values of mixed samples in Table 2 with errors of a few percent.

—References—

(1) T.Suzuki, K.Goda, N.Suzuki, J. Nucl. Sci. Technol., **30** [10] 1071 (1993).
(2) T.Suzuki, K.Goda, N.Suzuki, H. Ohno, J. Nucl. Sci. Technol., **32** [1] 75 (1995).