

# Equatorial Mounting Type Solar Heat Condenser

Takeo AIZAWA\* and Masayuki OGATA\*\*

## 1. Introduction

For solar heat condensing, a variety of methods have been proposed until today, but industrial utilization of solar heat would sometimes require a method to obtain relatively high temperature. The authors aim at studying an experimental model of solar heat condenser which has a parabolic cylinder mirror as the heat receiver and tracks the sun all day from sunrise to sunset to heat the water from 20°C to nearly 70°C. This paper deals with the following items.

- (1) Preparation of an experimental model of tracker and its test
- (2) Study of aberration caused by incident rays non-parallel to the parabolic axis

As the study of heat balance has not been thoroughly carried out, it will be dealt with in a future paper.

## 2. Elements for tracking

For tracking the sun, two elements of motion need to be taken into account. One is of the sun's daily motion (diurnal motion), that is one rotation a day. The other is of the vertical motion according to the seasonal change of the sun's altitude. Figure 2-1 shows how the sun's altitude changes seasonally. This change is owing

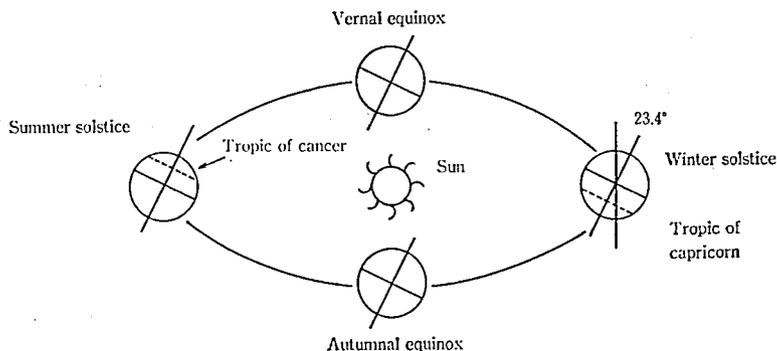


Fig. 2-1 Four seasons and altitude of the sun

\* 理工学部機械工学科教授 冷凍工学, 機構学

\*\* 理工学部機械工学科助手 流体工学

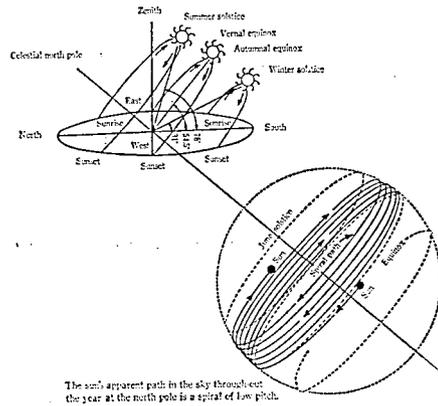


Fig. 2-2 The noon sun's altitude at Tokyo

to the fact that the plane of the equator is inclined  $23.4^\circ$  with the plane of the ecliptic. It is understandable that the direction of sun's rays at a given place on the earth changes seasonally. Figure 2-2 shows the altitude of the noon sun in Japan for each season. A tracker must follow the sun's motion of above-mentioned two elements.

### 3. Method of tracking

There are two kinds of methods for tracking the sun.

#### (1) Seeking method

Angular diameter of the sun is 32 minutes. So, if a heat receiver is linked with a photoelectric tube which follows the sun's center correctly, the receiver could be kept facing to the sun. But this mechanism will be rather intricate and costs high for practical use. And there is one more weak point that, once the weather become cloudy or rainy, the photoelectric tube stops seeking the sun, and when the weather recovers, readjustment of its position is required.

#### (2) Equatorial method

If an equatorial has the known sun's diurnal motion and seasonal change of the sun's altitude built-in, it is able to track the sun without interruption caused by the change of weather. It was, therefore, decided to adopt this method for the experimental model.

### 4. Principle of equatorial

Figure 4-1 shows the relation of equatorial to the earth, where  $N$  and  $S$  are the North Pole and the South Pole respectively, and  $O$  is the earth's center. Then,  $P$  is a point on the earth's surface (latitude:  $\theta$ ).  $P-N'$  is the equatorial's axis, which is in parallel to the earth's axis  $N-S$ . The included angle between  $P-N'$  and the horizon is  $\theta$ .

Now a rotation can be supposed on the axis  $P-N'$  to the same direction and at

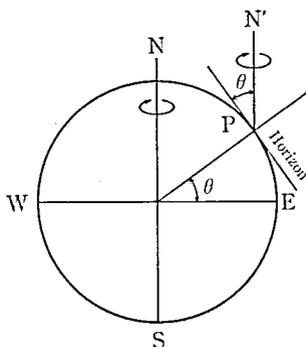


Fig. 4-1 Principle of equatorial

the same rate as the earth's rotation. If the rotation were observed from the sun, it would be regarded to occur at the earth's center, because, in comparison with the distance between the sun and earth, the earth's radius is so small that it can be disregarded. This means that the sun's rays at any point on the earth are parallel.

#### 5. Preparation of experimental equatorial

As the earth rotates on its axis once per 24 hours (1,440 minutes) in respect to the sun, the equatorial's shaft need to revolve at the rate of  $1/1,440$  rpm. To keep the rate precisely constant, a synchronous motor was applied to as the prime mover. To obtain the rate from the prime mover of  $50 \sim 1,500$ , the reduction ratio need to be precisely  $1/2,160,000$ . Strictly speaking, however, the rate of the sun's diurnal motion is varies. But the variation is so small that it need not be taken into account in revolution of the shaft.

In the beginning, the authors looked for a synchronous motor of 100 W and 1,500 rpm as the prime mover. But it was not in market. As a synchronous motor of 30 W and 1,500 rpm is available instead, it was decided to reduce the size of parabolic mirror to  $0.5 \text{ m} \times 0.5 \text{ m}$  for the experiment.

Figure 5-1 shows the train of gears which were chosen from catalogues of various makers. A set of reduction gears ② was employed for easy handling in gear changing, repairing and so on. Up to the present, however, such a case has not happened.

Worm gears were employed because they give high reduction ratios and selflock to the shaft. Two steps of worm gears ③ and ④ were set, not only because they had to be chosen from market but also because a single set of large reduction ratio would make the efficiency very low. The efficiency of the train of gears is estimated  $10 \sim 15\%$  in total.

The heat receiver revolves very slowly. But it is desirable to guarantee sufficient

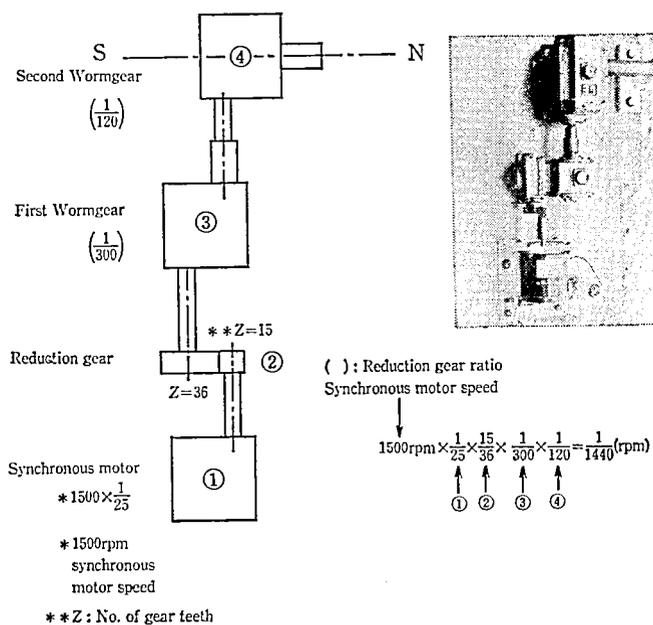


Fig. 5-1 Equatorial's reduction gears

rotary power. So, it was designed to weigh less than 20 kg using light metals.

Direction of the input shaft and output shaft of the second worm gear reducer ④ are at right angle to each other. To use the output shaft as the equatorial's shaft, the reducer must be tilted  $35.7^\circ$ , that is equal to the latitude of the testing site. The inclination allowance of this reducer is, however, specified as  $30^\circ$  by the maker. But, considering that there would be few problems because its revolution is very slow, it was decided to use it tilting  $35.7^\circ$  at the authors' own risk. There is, as a fact, no trouble up to the present. Troubles would occur, however, in case of using a larger-scale reducer and/or a longer running.

## 6. Preparation of heat receiver

The mirror of the heat receiver is made of polished 0.3 t stainless steel plate. To form a correct parabolic cylinder supported at several points, suitable supporting parts and reinforcing materials are required.

A copper pipe is used as the heat absorbing pipe, which is set on the focal line of the cylinder. For differential absorption of spectrum, a film of cupric oxide is formed on the pipe's surface, by oxidizing the copper surface with nitric acid and heating to  $300^\circ\text{C}$ . Figure 6-1 and Table 6-1 shows the heat receiver and the parts list respectively.

## 7. Connection of equatorial and heat receiver

The heat receiver was fixed on the equatorial's shaft. Figure 7-1 shows the

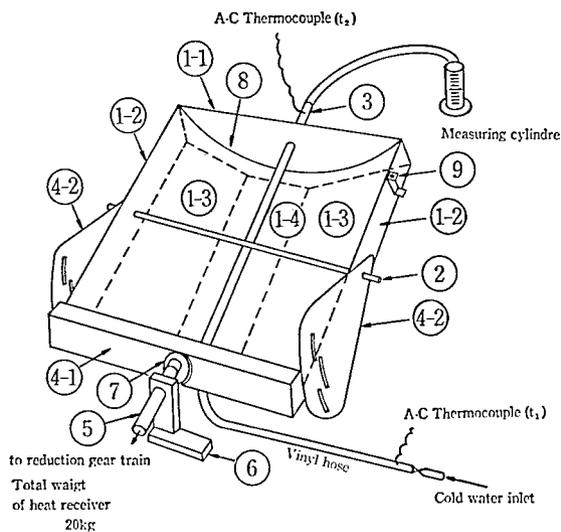


Fig. 6-1 Details of Heat receiver

Table 6-1 Parts list

No.	parts name	dimension	Q	material	note
1-1	outer frame	2×504×225	2	duralumin	
1-2	ditto	2×504×75	2	ditto	
1-3	ditto	2×504×220	2	ditto	
1-4	ditto	2×504×180	1	ditto	
2	center supportter	φ 8×560	1	SS 41	
3	heat absorbing pipe	φ 16 × φ 14 × 620	1	copper pipe	
4-1	heat receiver supporting frame	101 × 49.5 × 531	1	SS 41	
4-2	heat receiver supporting side plate	3.5 × 350 × 295	2	SS 41	
5	shaft	φ 30 × 180	1	SS 41	
6	bearing	138 × 133.5	1	SS 41	
7	reduction gear base plate (lower)	15 × 725 × 530	1	duralumin	
8	ditto (upper)	15 × 625 × 440	1	ditto	
9	shaft coupler	φ 79 × 38.5	1	SS 41	
10	parabolic mirror	0.3 × 690 × 500	1	polished stainless plate	
11	sight	2 × 75 × 25	1	duralumin	
12	connecting plate	2 × 100 × 60	20	ditto	

Q : quantity

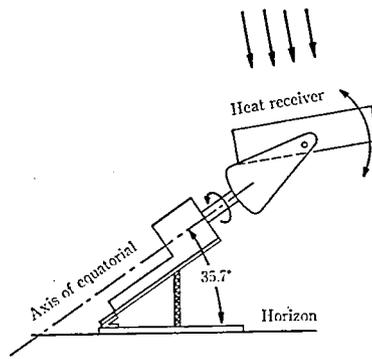


Fig. 7-1 Equatorial and Heat receiver

connection of the equatorial and heat receiver. Both are connected through heat receiver supporting side plate which is adjustable for changing the receiver's angle.

Two duralumin plates of 15 mm in thickness were prepared as the base plates. Both were connected with each other on sides by hinges. The equatorial with the heat receiver was set on the upper plate and a pair of levels intersecting at right angle were set on the lower plate. Keeping the lower plate horizontal, the upper plate was fixed tilting  $35.7^\circ$ .

The load is not expected to be well-balanced around the shaft, but the shaft will not rotate reversely due to selflock by worm gears.

#### 8. Setting of the model for test

The test site was selected near by the south side of the 9th Building of the Meisei University ( $35.7^\circ\text{N}$ ,  $139.4^\circ\text{E}$ ). In usual the direction of north is given by a magnetic compass, but, strictly speaking, a compass does not give the true north (geographical north) because the magnetic north pole is off the North Pole but stays in the northern part of Canada now. To obtain the true north, therefore, the magnetic north must be corrected by a magnetic declination. Magnetic declination varies from place to place on the earth. At the test site the declination is known to be  $6^\circ 40'$  W (1980) after the isogonic map published by the Geographical Survey Institute.

But in this case the compass was unstable probably because the test site is near a prime mover laboratory where various testing equipments and machines are placed. So, the experimental model was placed through checks of the spot of sunlight on the cross of the sight set on the receiver's frame. Figure 8-1 shows the total model.

#### 9. Adjustment of the model

The noon sun's altitude at Tokyo is  $78.0^\circ$  on the summer solstice,  $54.5^\circ$  on the vernal equinox and the autumnal equinox, and  $31.0^\circ$  on the winter solstice. During the three months between two of above dates, the altitude changes by  $23.4^\circ$ , that

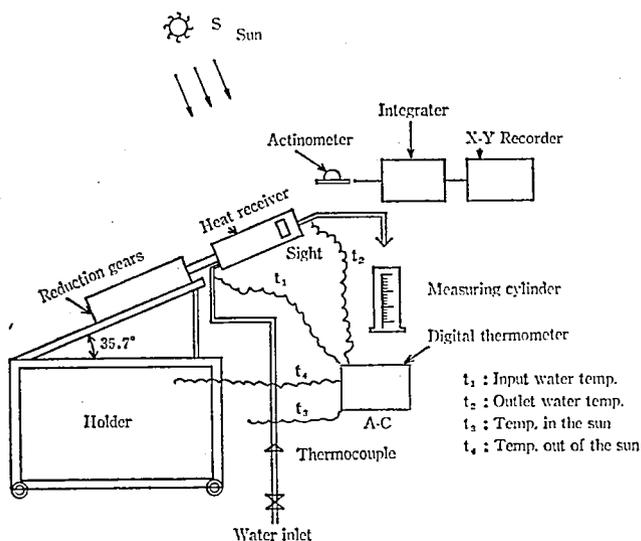


Fig. 8-1 Schematic total model

is about  $0.26^\circ$  a day. So, it is concluded that the angle of receiver might be corrected once a few days.

How correctly the receiver tracks the sun can be checked using the sight. If the spot of sunlight appears to the left of the center of cross, the equatorial is fast, and if to the right, it is slow. The deviation can be canceled by shifting the model so that the spot would come to the center. If the spot appears above the center, the receiver's angle is over, and vice versa. These vertical deviations can be also canceled by adjustment. Figure 9-1 shows the record of spot of sunlight on the cross. If checked every 10 minutes like this, correction can be easily made.

The length of sight is 71 mm. When the spot of sunlight in the sight is 2 mm off the center of cross, the angular deviation of incident rays from the parabolic axis is converted as about  $1.6^\circ$ . Allowance of the angular deviation will be discussed in the next section.

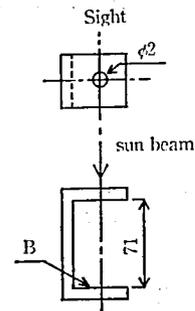
## 10. Path of incident rays

Figure 10-1 shows a parabolic mirror which is expressed as  $y^2=4px$ . If incident rays parallel to  $x$  axis reflect at  $A(x_1, y_1)$  on the mirror, they come to  $F$  (focus) on  $O-X$ . It is the same phenomenon that the rays reflect at  $A$  on a plane mirror which is tangent to the parabola. It is summarized that rays entering in parallel to  $x$  axis converge to  $F$  after reflection. The proof is shown in the appendix

Figure 10-2 shows the relation of the heat absorbing pipe to the parabolic cylinder mirror. The sun's rays in parallel to  $O-X$  are, therefore, concentrated to and absorbed by the pipe set on the focal line.

The equatorial may not always track the sun perfectly for some reasons.

A	11:20	11:30	11:40	11:50	12:00	12:10	12:20	12:30	12:40
B	⊕	⊕	⊕	⊕	⊕	⊕ (Reset)	⊕	⊕	⊕
C	-4.0	-1.5	1.0	4.0	6.0	7.5	10.5	12.5	15.5
D	G	P	G	G	G	G	G	G	G
E	16.8	17.0	18.0	17.5	17.2	17.0	16.5	16.2	17.0
A	12:50	13:00	13:10	13:20	13:30	13:40	13:50	14:00	14:10
B	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕
C	18.0	20.5	23.0	25.0	28.0	31.0	32.5	35.5	37.5
D	G	P	G	G	G	G	G	G	G
E	17.0	16.5	16.8	16.8	17.0	16.0	16.0	16.5	16.2
A	14:20	14:30	14:40	14:50	15:00	15:10			
B	⊕	⊕	⊕	⊕	⊕	⊕			
C	40.5	43.0	45.5	49.0	50.0	53.0			
D	G	G	G	G	G	G			
E	16.5	17.0	17.0	10.0	17.2	17.0			



- A : Time
- B : Image of the sun on the cross
- C : Angular deviation of heat receiver
- D : Judge of tracking accuracy (P : perfect, G : good, N : no good)
- E : Input power (W)

Fig. 9-1 Record of tracking (Nov. 22, 1983)

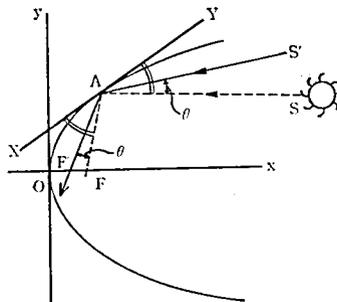


Fig. 10-1 Aberration caused by the sun's rays non-parallel to  $x$  axis

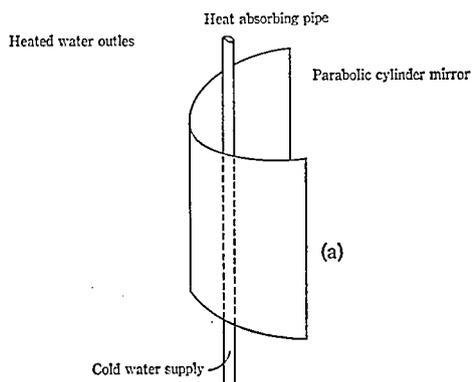


Fig. 10-2 Parabolic cylinder mirror and heat absorbing pipe

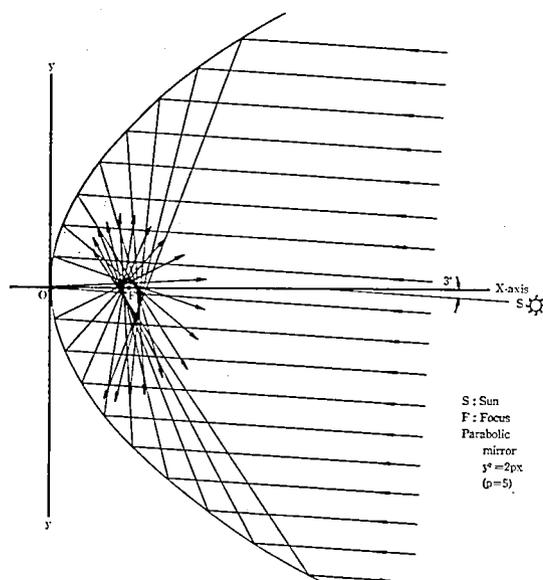


Fig. 10-3 Aberration caused by incident rays  $3^\circ$  deviated from the parabolic axis

Even if such cases happen, however, satisfactory efficiency in heat condensing is desirable to be kept. So, it is required to study how far the concentration will occur from the focus when incident rays are non-parallel to the axis.

The path of rays after reflecting on a parabolic mirror can be given as follows. As shown in Figure 10-1, entering onto the mirror in parallel to the parabolic axis O-X and reflecting at A, the sun's rays come to F. It is known that incidence angle is equal to reflection angle.

$$\angle XAF = \angle YAS$$

If the sun's rays come from the direction of  $S'$ , which is non-parallel to the axis by  $\theta$ ,

$$\angle XAF' = \angle YAS'$$

So, the rays come to  $F'$  at a distance from  $F$ . A caustic curve formed by  $A-F'$  would show the aberration of heat.

Figure 10-3, 10-4 and 10-5 show the caustic curves graphically drawn for angular

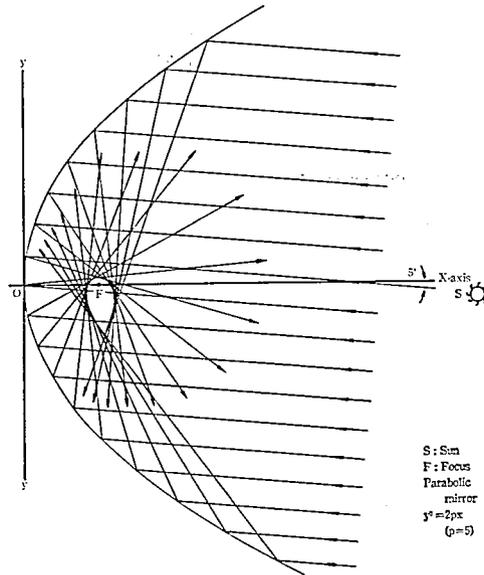


Fig. 10-4 Aberration caused by incident rays  $5^\circ$  deviated from the parabolic axis

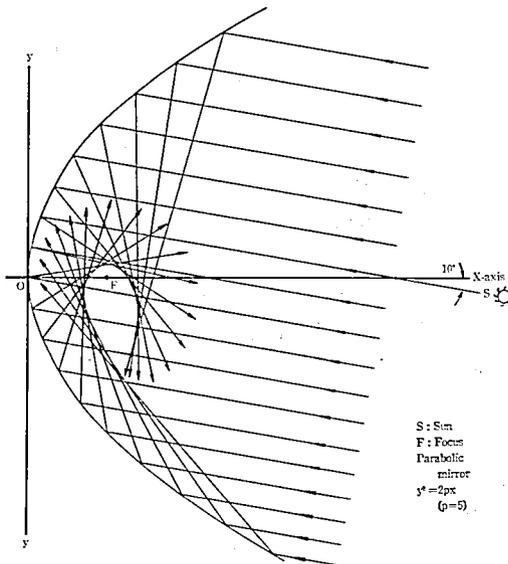


Fig. 10-5 Aberration caused by incident rays  $10^\circ$  deviated from the parabolic axis

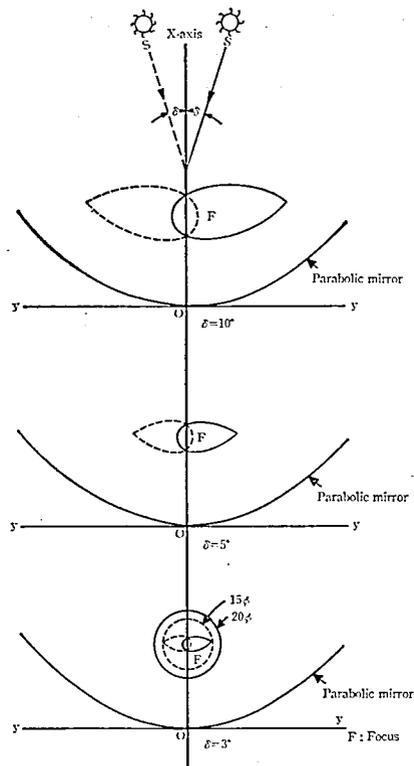


Fig. 10-6 Comparison of aberrations caused in the case of different angular deviation

deviation  $3^\circ$ ,  $5^\circ$  and  $10^\circ$  respectively. The parabola is expressed as  $y^2=2px$  ( $p=5$  cm).

Figure 10-6 shows comparison of the three envelope curves. In the case of  $\delta=3^\circ$ , for example, a heat absorbing pipe of  $\phi 15 \sim \phi 20$  would be good enough for efficient heat collecting.

## 11. Conclusion

(1) Though this experimental model was not necessarily composed of the best parts, the test made it sure that it works in success as a tracker. But, following problems still remains.

(a) Simplification of gear system should be studied, because it became sure that a simpler system could work well enough.

(b) Manual operation is practical enough for correcting the angle of heat receiver. So, it is desirable that a receiver is more simplified.

(c) Usage of a stepping motor should be studied for future largescale model.

(d) An equatorial is in need of a quick return function, because, after one day running, it must be revolved back quickly to become ready for the next morning.

(2) A study on the aberration of rays made it clear that a heat absorbing pipe of  $\phi 15 \sim \phi 20$  would be good for heat collection, when angular deviation of the incident rays from the parabolic axis is  $3^\circ$ .

### Acknowledgment

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### Appendix

A parabola is expressed as

$$y^2 = 4px$$

The tangent involving  $A(x_1, y_1)$  is given as

$$2yy' = 4px$$

$$y' = \frac{2p}{y_1}$$

$$(y - y_1) = \frac{2p'}{y_1}(x - x_1)$$

If  $y=0$ , the coordinate of  $X$  is given as

$$-y = \frac{2p}{y_1}(x - x_1)$$

$$-y^2 = 2p(x - x_1)$$

$$-4px = 2px - 2px_1 \quad \therefore x = -x_1 \quad (1)$$

The length of  $AF$  is

$$\sqrt{y_1^2 + (x_1 - p)^2} = \sqrt{4px_1 + (x_1 - p)^2} = x_1 + p \quad (2)$$

From (1),

$$\begin{aligned} \overline{XF} = x_1 + p \quad \therefore \overline{XF} = \overline{AF} \\ \therefore \angle X = \angle A \end{aligned} \quad (3)$$

Hence the rays from  $S$  reflect at  $A$  and come to  $F$ .