Mixed Education System and Inequality

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Abstract

We study a simple growth model in which public and private education coexist. Parents provide their children with either public or private education. Since the level of public education for a student is fixed, the government needs less revenue when the number of students attending public school declines, which lowers the tax rate. A lower tax rate is more beneficial for the parents who choose private education for their children than for those who choose public education. The converse is also true. This mechanism leads to multiple equilibria of educational choices by parents. The model can create interesting dynamics. In the early stage of development, public and private education coexist and income inequality grows. In the later stage, all parents may choose private education and inequality may shrink.

JEL classifications: I25, O15

Keywords: Public and Private Education, Human Capital Accumulation, Inequality.

1 Introduction

In most countries, education is largely provided by the government. It is widely believed that education is the key factor that alleviate poverty and that education has a positive externality. The public provision of education is strongly justified. Of course, it is possible to provide education privately, and many private schools exist. Furthermore, private education has become more widespread. According to the Education at a Glance 2018 report by the OECD, between 2010 and 2015, the share of private spending on primary to tertiary educational institutions increased by 11% on average across OECD countries, while the share of public sources decreased by about 1%. "While public funding still represents a large part of countries' investment in education, the role of private sources of funding is becoming increasingly prominent at some educational levels.(p.270)"

There is a vast literature on economic growth and education. Since Lucas (1988), researchers viewed human capital accumulation as one of main factors that affect economic growth and development. Other researchers applied this idea to various analyses. Among others, Glomm and Ravikumar (1992) compare the public and private education systems and examine their effects on economic

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development and inequality. In their model, all students receive public education under the public education system and all students receive private education under the private education system.

In this study, we analyze a growth model with a mixed education system in which public and private education exist, and examine its implications for economic development and inequality. Under this system, parents choose either public or private education for their children. Public education provides students with a fixed and uniform level of education. In contrast, parents can choose a level of private education. Parents with sufficient wealth and dissatisfaction with the level of public education will choose private education. Because the level of per capita public educational expenditure is fixed, the government needs to collect more revenue and raise the tax rate when more students attend public school. This change in the tax rate may affect parents' educational choices. Specifically, the rise in the tax rate is more harmful to those who choose private education than those who choose public education, and thus induces more parents to choose public education. In sum, the increase in the number of parents who choose public education for their children induces other parents to choose public education, and vice versa. Consequently, multiple equilibria may occur. The model has two types of agents: high-income and low-income. The mechanism above applies to each type of agents, and three types of equilibria can emerge: one in which all parents choose public education, one in which all parents choose private education, and one in which high-income parents choose private education and low-income parents choose public education.

We also examine the dynamic implications of the model. Because the per capita educational cost is constant, if the number of students who attend public school does not change, economic growth reduces the burden of public education. An increase in aggregate production makes it possible to lower the income tax rate to finance the total cost of public education. As we note above, the decline in the income tax rate induces parents to choose private education. That is, in a rich country, parents tend to choose private education. This effect can generate an interesting dynamic process. Suppose that an economy is unequal and that high-income individuals receive private education and low-income individuals receive public education in the early stage of development. As the economy grows, the income tax rate declines. When the economy reaches a certain level of development, low-income individuals may start to choose private education. Afterward, the economy realizes a high level of development and equality. In another scenario, the economy may be trapped at a low level of development and remain unequal. The simplified setup of the model enables us to analyze the complicated development process.

Some studies deal with mixed education systems. Stiglitz (1974) points out the difficulty of applying the median voter theorem to an economy with a mixed education system. Epple and Romano (1996) find the conditions under which a majority voting equilibrium exists. Glomm and Ravikumar (1998) show a specific setup that yields a voting equilibrium. These models are static[1]. Cardak

[1]See also Bearse, Glomm and Patterson (2005)

(2004) constructs a dynamic model with a mixed education system and examines it by using simulations[2]. In contrast, the model in our study is dynamic and we study it analytically.

The remainder of the paper proceeds as follows. Section 2 describes the basic setup of the economy. Section 3 considers households' decisions. Section 4 illustrates the equilibria. Section 5 analyzes dynamics of the economy. Section 6 concludes.

2 Model

Consider an over-lapping generations economy in which individuals live for two periods. The economy has two types of individuals, high-income and low-income, which we distinguish according to their level of their income. The numbers of high- and low-income individuals in period t are H_t and L_t, respectively, where $H_t + L_t = 1$. Each individual has a single parent and a single child, so the population is constant over time. In the first period, childhood, each individual receives education according to the parent's choice. Young individuals make no economic choice. In the second period, adulthood, each individual has one child and one unit of time to allocate between labor and leisure. The individual's income is allocated to consumption and may be allocated to expenditure on the child's education.

The economy has two systems of education. One is a private school at which the parent can freely choose the level of education and must pay the corresponding education cost. The other is a public school that provides a uniform level of education to students. An income tax finanes the public school, so the parents of the children attending the public school do not pay tuition. Parents must choose between the public and private schools and make their children attend.

The utility function of an individual of generation t is

$$U_t = (1 - \beta_1 - \beta_2) \ln (1 - n_{t+1}) + \beta_1 \ln c_{t+1} + \beta_2 \ln e_{k,t+1}, \tag{1}$$

where n_{t+1} is the fraction of time an individual of generation t spends working in period $t+1$, c_{t+1} is the consumption of an individual of generation t in period $t+1$, $\beta_1 > 0$ is the weight attached to her consumption, and $\beta_2 > 0$ is the relative weight of the level of the child's education. $e_{k,t+1}$ is the level of education that the child receives, where $k = r, u$ represents the type of education, either private or public. $\beta_2 \ln e_{k,t+1}$ reflects altruism factor which is referred to as joy of giving. We assume that $\beta_1 + \beta_2$ is less than 1 and that $1 - \beta_1 - \beta_2$ is the weight attached to leisure. The level of her child's education depends on her education choices.

$$e_{k,t+1} = \begin{cases} e_u & \text{public education} \\ e_{r,t+1} & \text{private education} \end{cases},$$

[2]See also Arcalean and Schiopu (2010) and de la Croix (2013).

where e_u is the constant level of public education[3]. We assume that the level of public education e_u is sufficiently small so that the income tax rate is smaller than one. When a parent decides to make her child receives private education, she chooses the level of education, $e_{r,t+1}$. The budget constraint[4] of an individual of generation t:

$$c_{t+1} + e_{r,t+1} = (1 - \tau_{t+1})\, y_{t+1}, \tag{2}$$

where y_{t+1} is income of a member of generation t, which depends on the level of education $e_{k,t}$ she received in her youth and the fraction of time spent on working, n_{t+1}. We adopt a specific functional form of y_{t+1} as

$$y_{t+1} = A e_{k,t}^{\alpha_1} n_{t+1}^{\alpha_2} \tag{3}$$

where $A > 0$, $0 < \alpha_1, \alpha_2 < 1$.

Public education is financed by a proportinal tax, τ_{t+1}, on income. The government budget constraint is

$$N_{t+1} e_u = \tau_{t+1} Y_{t+1}, \tag{4}$$

where N_{t+1} is the number of students who attend the public school and Y_{t+1} is aggregate income. Parents choose either public school or private one to make their children enter, so N_{t+1} is a variable. The public school offers a basic level of education which is necessary to live in the society, so e_u is a constant. The government cannot control aggregate income, so the tax rate, τ_{t+1}, is adjusted to equate expenditure and revenue every period.

3 Household Decision

A parent chooses leisure, consumption and a education system for her child. First, we consider the optimization of the parent who chooses the private school.

[3]The assumption that the per capita public educational expenditure is constant is crucial in the model. It reflects following considerations. First, education has its minimum and requisite contents, such as literacy and numeracy. Teaching alphabets from A to S is useless. Second, the government must provide students with minimum knowledge and skills which are necessary for daily life and working in the economy, but is not required to provide additional and higher knowledge and skills. In other words, a range of choices in the level of public education is narrow. The assumption that the per capita public educational expenditure is constant is an extreme way to reflect these considerations. Needless to say, in reality, it is possible that the governments of poor economies can not collect sufficient revenue to provide students with the minimum level of education. Therefore, the analysis of the model applys to economies which are not so poor.

[4]The supply side of the model is extremely simplified. Production needs efficiency labour as the sole factor. And the technology is linear, that is, one unit of efficiency labour is converted into one unit of consumption goods or education. Thus, we can interpret equation (2) in two ways. First, a parent produce consumption goods, $c_{i,t+1}$, and private education, $e_{r,t+1}$, using her efficiency labour. Second, there are competitive firms with linear technology which produce consumption goods or education in the economy. A parent supplies her efficiency labour and earns the same wage as the labour, because firms earn zero profit due to competition. And then she buys consumption goods and private education according to her budget constraint. In both cases, a parent's total expenditure is equal to the amount of her aftertax efficiency labour.

3.1 When a parent chooses private school

We can rewrite the utility function (1):

$$U_t^r = (1-\beta_1-\beta_2)\ln(1-n_{t+1})+\beta_1\ln[(1-\tau_{t+1})\,y_{t+1}-e_{r,t+1}]+\beta_2\ln e_{r,t+1}, \quad (5)$$

where r denotes private school. The first-order condition for a maximum U_t^r with respect to the level of education $e_{r,t+1}$ leads to:

$$e_{r,t+1} = \frac{\beta_1}{\beta_1+\beta_2}(1-\tau_{t+1})\,y_{t+1}. \quad (6)$$

Education spending by a parent is a constant fraction of her disposable income.

Next, the first-order condition with respect to the fraction of time spent working n_{t+1} leads to:

$$\frac{\frac{\partial y_t}{\partial n_{t+1}}}{y_t} = \frac{1-\beta_1-\beta_2}{\beta_1+\beta_2}\frac{1}{1-n_{t+1}}. \quad (7)$$

Substituting the production function (3) into (7) leads to the optimal fraction of time spent working:

$$n_r = \frac{\alpha_2}{\left(\frac{1-\beta_1-\beta_2}{\beta_1+\beta_2}+\alpha_2\right)} \quad (8)$$

We find that when the parents who choose the private school the optimal fraction of time spent working is constant over time.

3.2 When a parent chooses public school

Now, consider the optimization of the parents who choose the public school. The gorvenment provides the common level of education to students. Hence, a parent does not choose an individual education level for her child. We can rewrite the utility function (1):

$$U_t^u = (1-\beta_1-\beta_2)\ln(1-n_{t+1})+\beta_1\ln(1-\tau_{t+1})\,y_{t+1}+\beta_2\ln e_u, \quad (9)$$

where u denotes public school. The first-order condition for a maximum U_t^u with respect to the fraction of time spent working n_{t+1} gives us:

$$n_u = \frac{\alpha_2}{\left(\frac{1-\beta_1-\beta_2}{\beta_1}+\alpha_2\right)}. \quad (10)$$

This is constant and smaller than the fraction of time spent working when a parent chooses a private school, that is,

$$n_u < n_r.$$

The reason for this difference is as follows. Parents choose the time spent working so as to equate marginal benefit of working with marginal cost of working.

When the parents choose the public school marginal benefit of working is *only* due to the increase of consumption. On the other hand, when they choose the private school marginal benefit of working is due to the increase of consumption *plus* the increase of education level. Marginal costs of working are the same between the two. Thus, a parent works more when she sends her child to a private school than to a public one.

Given the level of education that they received when young, the incomes of the parents who choose the private school is higher than that of the parents who choose the public school because of the difference of working time. That is,

$$y^u_{t+1} < y^r_{t+1}.$$

3.3 Educational Choice

A parent chooses a private (public) school to make her child attend if her utility from a private school, U^r_t, is greater (smaller) than her utility from a public school, U^u_t. Substituting both (6) and (8) into (5) yields:

$$\begin{aligned} U^r_{j,t} &= (1-\beta_1-\beta_2)\ln(1-n_r) \\ &\quad + \ln\left[\left(\frac{\beta_1}{\beta_1+\beta_2}\right)^{\beta_1}\left(\frac{\beta_2}{\beta_1+\beta_2}\right)^{\beta_2}\left[(1-\tau_{t+1})\,y^r_{j,t+1}\right]^{\beta_1+\beta_2}\right], \end{aligned} \tag{11}$$

where $j = r, u$ represents the type of education the parent received when young. Similarly, substituting (10) into (9) yields:

$$U^u_{j,t} = (1-\beta_1-\beta_2)\ln(1-n_u) + \beta_1\ln\left[(1-\tau_{t+1})\,y^u_{j,t+1}\right] + \beta_2\ln e_u. \tag{12}$$

We define a function Φ by combining equations (11) and (12):

$$\Phi_{j,t} \equiv U^r_{j,t} - U^u_{j,t} \tag{13}$$

$$\begin{aligned} &= (1-\beta_1-\beta_2)\ln\frac{1-n_r}{1-n_u} + (\beta_1+\beta_2)\ln y^r_{j,t+1} \\ &\quad + \beta_2\ln(1-\tau_{t+1}) - \beta_1\ln y^u_{j,t+1} - \beta_2\ln e_u + \ln Z, \end{aligned} \tag{14}$$

where the constant Z is defined as

$$Z \equiv \left(\frac{\beta_1}{\beta_1+\beta_2}\right)^{\beta_1}\left(\frac{\beta_2}{\beta_1+\beta_2}\right)^{\beta_2}.$$

The value of $\Phi_{j,t}$ represents utility difference between private and public education. If it is positive for a parent, she chooses to make her child receive private education. We can see that a rise in τ_{t+1} decreases $\Phi_{j,t}$. If a parent makes her child receive private education, she must decrease expenditures on consumption and education in response to a rise in the income tax rate. She sustains a utility loss. By contrast, a parent who makes her child receive public education must decrease expenditure only on consumption when the tax rate rises. The level of public education does not change. So her utility loss is smaller than one which

she sustains when she chooses private education. As a result, the rise in the tax rate decreases the utility difference. The value of τ_{t+1} depends on the number of students who receive public education. In the next section, we analyze $\Phi_{j,t}$ and find the equilibrium number of students who receive public and private education.

4 Equilibrium

We assume that all rich parents received private education when young and that all poor parents received public education when young in the initial period. In other words, the economy is unequal.

We denote the number of students who receive public education and whose parents received private education by m^d, and the number of students who receive private education and whose parents received public education by m^u, where $0 \leq m^d \leq H_t$ and $0 \leq m^u \leq L_t$. That is, m^d and m^u represent the number of children who receive the different type of education from their parents. We can interpret them as intergenerational mobility: m^d as downward mobility and m^u as upward mobility. Then, the number of students who receive public education, N_{t+1}, is written as

$$N_{t+1} = m^d_{t+1} + (L_t - m^u_{t+1}). \tag{15}$$

And aggregate production can be written as

$$Y_{t+1} = (H_t - m^d_{t+1})y^r_{r,t+1} + m^d_{t+1}y^u_{r,t+1} + (L_t - m^u_{t+1})y^u_{u,t+1} + m^u_{t+1}y^r_{u,t+1}. \tag{16}$$

Therefore, the income tax rate is determined as

$$\tau_{t+1} = \frac{(m^d_{t+1} - m^u_{t+1})e_u + L_t e_u}{H_t y^r_{r,t+1} + L_t y^u_{u,t+1} - m^d_{t+1}(y^r_{r,t+1} - y^u_{r,t+1}) + m^u_{t+1}(y^r_{u,t+1} - y^u_{u,t+1})}. \tag{17}$$

$y^k_{j,t+1}$ is production by a parent, the type of education that she received when young is denoted by $j = r, u$. It is clear that $\partial\tau_{t+1}/\partial m^d_{t+1} > 0$ and $\partial\tau_{t+1}/\partial m^u_{t+1} < 0$. In words, a rise in the number of students who receive public education raises the income tax rate.

Now, we can analyze the parental choice between private and public education and derive the equilibrium number of students. In the initial period, there are H_t high-income parents, who received private education when young, and m^d_{t+1} of them decide to make her children receive public education. There are L_t low-income parents, who received public education when young, and m^u_{t+1} of them decide to make her children receive private education. We need to determine the values of m^d_{t+1} and m^u_{t+1} in equilibrium. If $\Phi_{r,t} > (<)\ 0$ in equilibrium, it is not rational for a high-income parent to choose public (private) education for her child, and thus m^d_{t+1} must be equal to zero (H_t). Similar consideration can be applied to the choice by a low-income parent.

In order to find candidates of equilibrium, we depict $\Phi_{j,t} = 0, j = r, u$ curves on (m^d_{t+1}, m^u_{t+1}) plane. The functions $\Phi_{r,t} = 0$ and $\Phi_{u,t} = 0$ have the

same functional form and different constants. So they do not intersect. The difference between $\Phi_{r,t}$ and $\Phi_{u,t}$ is in the type of education that the parent received when young, e_{rt} and e_u. Since $e_{rt} > e_u$, $\Phi_{r,t}$ is larger than $\Phi_{u,t}$. They have the derivative,

$$\frac{dm^d_{t+1}}{dm^u_{t+1}} = -\frac{\partial\tau}{\partial m^u_{t+1}} \Big/ \frac{\partial\tau}{\partial m^d_{t+1}} > 0,$$

so they have positive slopes on (m^d_{t+1}, m^u_{t+1}) plane. From (14) and (17), we can get

$$\frac{\partial\Phi_{r,t}}{\partial m^d_{t+1}} < 0 \quad \text{and} \quad \frac{\partial\Phi_{u,t}}{\partial m^u_{t+1}} > 0.$$

The positions of $\Phi_{r,t} = 0$ and $\Phi_{u,t} = 0$ curves depend on parameter values. Fig 1 and 2 are examples. In the upper area of $\Phi_{r,t} = 0$ curve, $\Phi_{r,t}$ is negative so $m^d_{t+1} = H_t$. In the lower area, $\Phi_{r,t}$ is positive so $m^d_{t+1} = 0$. In the left area of $\Phi_{u,t} = 0$ curve, $\Phi_{u,t}$ is negative so $m^u_{t+1} = 0$. In the right area, $\Phi_{u,t}$ is positive so $m^u_{t+1} = L_t$. There are three candidates for equilibrium. Point e corresponds to downward-mobility equilibrium. Children born of parents who received private education receive public education. Point f corresponds to no-mobility equilibrium. Children receive the same types of education as their parents. Point g corresponds to upward-mobility equilibrium. Children born of parents who received public education receive private education.

5 Dynamics

There are three types of dynamic process without mobility. First, all students receive public education. Second, all students receive private education. Third, students from rich households receive private education and students from poor households receive public education.

5.1 All students receive public education

Because both the level of public education, e_u, and the effort level of an agent who choose public education, n_u, are constant, production level,

$$y^u_{u,t+1} = Ae^{\alpha_1}_u n^{\alpha_2}_u, \tag{18}$$

is also constant. So inequality in income vanishes once all students receive public education.

In this case, privately-educated parents will not exist next period, that is $H_{t+1} = 0$, and thus $\Phi_{r,t} = 0$ curve does not matter on (m^d_{t+1}, m^u_{t+1}) plane. Where the $\Phi_{u,t} = 0$ curve intersects with the horizontal axis determines which type of equilibrium exists. As noted above, the $\Phi_{u,t} = 0$ curve shifts as productivity changes. When productivity is low, only no-mobility equilibrium exists. As productivity rises, upward-mobility equilibrium emerges. When productivity rises further, no-mobility equilibrium disappears. Then, all parents choose private education for their children.

Proposition 1 *When all parents choose public education for their children in a period (i.e. the downward-mobility equilibrium materializes), all descendants will have the same level of income and equality of income will be realized from next period onward.*

5.2 All students receive private education

The production level of an individual who received private education and choose private education for her child is given by

$$y^r_{i,t+1} = A\left(\frac{\beta_1}{\beta_1+\beta_2}y^r_{i,t}\right)^{\alpha_1} n_r^{\alpha_2}, \quad i = r,\ u. \tag{19}$$

No one receives public education, so the income tax rate is zero. We can see from (19) that the production levels of both high-income and low-income households converge to

$$y^r_{ss} = \left[A\left(\frac{\beta_1}{\beta_1+\beta_2}\right)^{\alpha_1} n_r^{\alpha_2}\right]^{\frac{1}{1-\alpha_1}} \tag{20}$$

in the steady state. As a result, inequality in income vanishes in the long run.

In this case, there are no households whose child receive public education, $L_t = 0$, and thus $\Phi_{u,t} = 0$ curve does not matter on (m^d_{t+1}, m^u_{t+1}) plane and $\Phi_{r,t} = 0$ curve intersects the vertical axis at $m^d_{t+1} > 0$. Both an increase in $y^r_{i,t+1}$ and a rise in productivity shift $\Phi_{r,t} = 0$ curve upward. Therefore, downward-mobility equilibrium will disappear over time.

Proposition 2 *When all parents choose private education for their children in a period (i.e. the upward-mobility equilibrium materializes), income inequality will diminish gradually from next period on and vanish in the long run.*

5.3 The high-income choose private education and the low-income choose public education

In no-mobility equilibrium, production level of the households which choose public education is constant, $y^u_{u,t+1} = Ae_u^{\alpha_1} n_u^{\alpha_2}$. And production level of the households which choose private education is given by

$$y^r_{r,t+1} = A\left[\frac{\beta_1}{\beta_1+\beta_2}(1-\tau_t)y^r_{r,t}\right]^{\alpha_1} n_r^{\alpha_2}, \tag{21}$$

where the income tax rate is determined as

$$\tau_t = \frac{L_{t-1}e_u}{H_{t-1}y^r_{r,t} + L_{t-1}y^u_{u,t}}$$

in this case. Thus, equation (21) is an increasing and concave function of $y^r_{r,t}$ and has a steady state $y^r_{r,t+1} = y^r_{r,t} = y^r_{r,ss}$. Along the development path, $y^r_{r,t}$ grows toward its steady state and $y^u_{u,t}$ is constant. As a result, inequality between $y^r_{r,t}$ and $y^u_{u,t}$ expands.

Proposition 3 *Under the mixed educatin system, income inequality expands until the economy reaches its steady state.*

From (18), (19) and (21), we can compare the aggregate level of production. Under the public education system, all individuals spend less money on education and less time on work than under the private education system. The aggregate production under the mixed education system is an intermediate level. We can conclude:

Proposition 4 *The aggregate level of production is the largest under the private education system, and the smallest under the public education system.*

5.4 Development Process

In the previous subsection, we assume that the economy remains in no-mobility equilibrium. But mobility can occur. As $y^r_{r,t}$ increases, $\Phi_{r,t} = 0$ and $\Phi_{u,t} = 0$ curves move upward. Exogenous improvement in technology A also moves both upward. Suppose that, in a level of development, the economy is in the situation depicted in Fig 1 and no-mobility equilibrium (f) is realized. As the economy develops, $\Phi_{r,t} = 0$ and $\Phi_{u,t} = 0$ curves move upward. As a result, the situation may change into Fig 2. Then, no mobility is no longer an equilibrium. Some parents must change their behaviors. The more probable result is that upward mobility occurs. That is, parents who received public education choose private education for their children. After that, the economy is under the private education system and converges to the steady state with the high aggregate level of production. Along this development path, the economy exhibits kuznets curve. That is, while the economy is under the mixed education system, income inequality expands. After the economy moves into the private education system, inequality shrinks and will vanish in the long run.

6 Conclusion

In this paper, implications of the mixed education system are examined. Parents choose either public or private education for their children. The government collects income taxes and provides public education. The income tax rate is adjusted through the government budget constraint, and the adjustment changes parental decisions. In the long run, the economy may be trapped in the unequal and low production steady state in which public and private education coexist. Also it is possible that the economy arrives at the equal and high production steady state in which all students receive private education. The model in this paper is highly stylized, so that we can clarify the mechanism of economic development and educational choices. This model may serve to understand the upward trend in private education mentioned in the introduction.

Some results derived in the paper may seem unrealistic or extreme. For example, no country has abolished public education in the real world. So it might be meaningful to extend the model to contain more realistic distribution of income, rather than two-class distribution.

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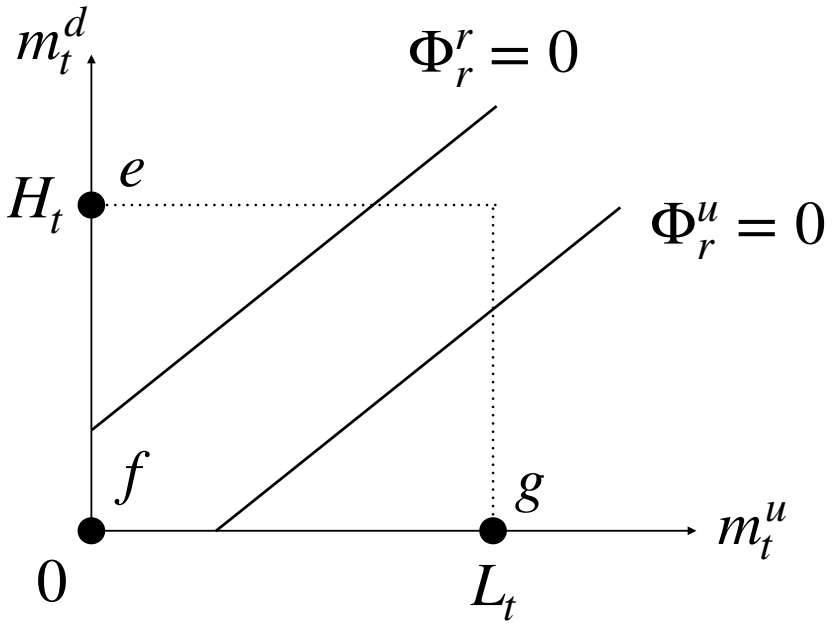

Figure 1: Utility differences: three equilibria

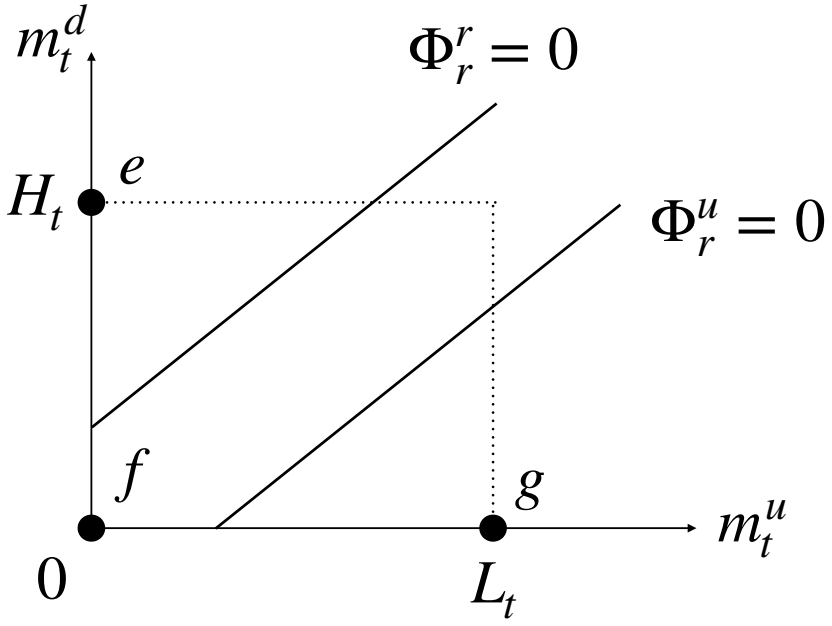

Figure 2: Utility differences: two equilibria